

	lun. 13/3	mar. 14/3	mer. 15/3	jeu. 16/3	ven. 17/3
GMT+01					
9am					
10am	9:30 - Coffee & Registration 10 - 10:50 Hannah Markwig	9:30 - Coffee & pastries 10 - 10:50 Tamas Hausel	9:30 - 10:20 John Hamad 10:30 - Coffee & pastries 11 - 11:50 Bertrand Eynard	9:30 - Coffee & pastries 10 - 10:50 Norman Do 11 - 11:50 Bram Petri	9:30 - 10:20 Konstanze Rietsch 10:30 - Coffee & pastries 11 - 11:50 Emmanuel Letellier
11am	11 - 11:50 Rahul Pandharipande	11 - 11:50 Hugo Parlier	11 - 11:50 Bertrand Eynard	11 - 11:50 Bram Petri	11 - 11:50 Emmanuel Letellier
12pm			12p - 12:50p Motohico Mulase		12p - 12:50p Stavros Garoufalidis
1pm					
2pm	1:45p - 2:35p Jeremy Guere	1:45p - 2:35p Martin Moeller		1:45p - 2:35p Alexander Buryak	
3pm	2:45p - 3:35p Felix Janda	2:45p - 3:35p Anton Zorich		2:45p - 3:35p Omid Amini	
4pm	3:40p - Coffee break 4:10p - 5p Aaron Pixton	3:40p - Coffee break 4:10p - 5p Don Zagler		3:40p - Coffee break 4:10p - 5p Dino Lorenzini	
5pm					
6pm		6p - 7:30p Cocktail at IHP/Poster session			
7pm					

## IHP Workshop, March 13th-17th, 2017

### *Enumerative geometry*

## Talks

OMID AMINI (ENS PARIS)

*Limits of linear series on degenerating families of curves: a fruitful example of interactions between combinatorics and algebraic geometry*

I will discuss my recent and ongoing work on getting a general theory of limit linear series for curves, generalizing the pioneering work of Eisenbud and Harris in the eighties. Eisenbud-Harris theory explains the degenerations of linear series as smooth curves approach singular curves of compact type, and has been used in several applications to explain the geometry of the moduli space of stable curves. However, stable curves of compact type provide only divisorial information on the moduli space, and it is therefore natural to extend the theory to all stable curves.

I will present two different approaches: a purely combinatorial one encoded in the theory of slope structures on metric graphs, and a fundamentally new approach developed in an ongoing joint work with EDUARDO ESTEVES (IMPA), based on the idea of degenerating points.

The building blocks in both these frameworks are a mixture of ideas from graph theory, discrete geometry, and algebraic geometry, and lead to applications in algebraic and arithmetic geometry.

ALEXANDER BURYAK (ETH ZÜRICH)

*Integrable hierarchies and tautological relations.*

I will present a conjectural system of relations in the tautological cohomology of the moduli space of curves. These relations naturally appear in the study of two different constructions of integrable hierarchies associated to cohomological field theories. The relations have surprising consequences, in particular, they give a new formula for the top Chern class of the Hodge bundle on the moduli space of curves. I will also discuss an evidence for the conjecture. The talk is based on a joint work in progress with JÉRÉMY GUÉRÉ and PAOLO ROSSI.

NORMAN DO (MONASH UNIVERSITY)

*Towards the topological recursion for double Hurwitz numbers*

Single Hurwitz numbers enumerate branched covers of the Riemann sphere with specified genus, prescribed ramification over infinity, and simple branching elsewhere. Among the many facets of their rich structure, they arise as intersection numbers on moduli spaces of curves and are governed by the topological recursion. Double Hurwitz numbers are defined analogously, but with prescribed ramification over both zero and infinity. Goulden, Jackson and Vakil have conjectured that double Hurwitz numbers also arise as intersection numbers on moduli spaces. In this talk, we propose the conjecture that double Hurwitz numbers are also governed by the topological recursion and consider the ramifications. Joint work with ANUPAM CHAUDHURI, MAX KAREV and DANILO LEWANSKI.

BERTRAND EYNARD (IPHT CEA SACLAY & CRM MONTRÉAL)

*Weighted Hurwitz numbers and topological recursion II.*

Weighted Hurwitz numbers count the number of branched cover of the Riemann sphere, with a given topology, and with a weighting dependent on the ramification profiles. The generating function is known to be a KP tau function. There are plenty of exact formulas for the generating series graded by the degree of the branched cover. However, often one

is interested in grading by the topology, and the 2 types of expansion are very non-trivially related. We show that the topological expansion is governed by topological recursion, with a very simple spectral curve. We give a sketch of a proof, relying on quantum curves, on Christoffel-Darboux relations, and determinantal formulas. Then we mention some consequences of topological recursion, like generalizations of ELSV formulas and identities satisfied by weighted Hurwitz numbers.

STAVROS GAROUFALIDIS (GEORGIA TECH)

*Counting surfaces in 3-manifolds and the 3d index.*

The 3d index of a 3-manifold with torus boundary (introduced by Dimofte-Gaiotto-Gukov) is a collection of  $q$ -series with integer coefficients parametrized by an integer homology class of the boundary. We will discuss the case of the trivial homology class, and the coefficient of  $q$  of the above series and identify the invariant with the counting of incompressible/Heegaard genus 2 surfaces. Joint work with subsets of NATHAN DUNFIELD, CRAIG HODGSON, NEIL HOFFMAN, HENRY SEGERMAN and HYAM RUBINSTEIN.

JÉRÉMY GUÉRE (HUMBOLDT UNIVERSITÄT)

*Enumerative theory of complex curves from singularities*

In 2007, Fan, Jarvis, and Ruan studied intersection theory on a moduli space of curves associated to a quasi-homogeneous polynomial singularity. Based on insights from Witten, it is viewed as a GIT variation of Gromov-Witten theory of hypersurfaces in weighted projective spaces. It is called the quantum singularity theory (or FJRW theory). More precisely, the moduli problem deals with a generalization of spin curves and the intersection theory on it is defined via an algebraic cycle based on the concept of matrix factorizations. In my talk, I will briefly describe FJRW theory and show how the same ideas lead us to a K-theoretic version as well. I will also discuss how classical results on Koszul cohomology yield explicit computations, even in positive genus.

JOHN HARNAD (CRM MONTRÉAL)

*Weighted Hurwitz numbers and topological recursion I.*

Multiparametric families of hypergeometric  $\tau$ -functions of KP or Toda type serve as generating functions for weighted Hurwitz numbers, providing weighted enumerations of branched covers of the Riemann sphere. The theory is placed within the framework of topological recursion, with the Baker function at  $t = 0$  shown to satisfy the quantum spectral curve equation, whose classical limit is rational. A basis for the space of formal power series in the spectral variable is generated that is adapted to the Grassmannian element associated to the  $\tau$ -function. Multicurrent correlators are defined in terms of the  $\tau$ -function and shown to provide an alternative generating function for weighted Hurwitz numbers. Fermionic VEV representations are provided for the adapted bases, pair correlators and multicurrent correlators. Choosing the weight generating function as a polynomial, and restricting the number of nonzero “second” KP flow parameters in the Toda  $\tau$ -function to be finite, implies a finite rank covariant derivative equation with rational coefficients satisfied by a finite “window” of adapted basis elements. The pair correlator is shown to provide a Christoffel-Darboux type finite rank integrable kernel, and the WKB series coefficients of the associated adjoint system are computed recursively, leading to topological recursion relations for the generators of the weighted Hurwitz numbers. Based in part on joint work with MATTHIEU GUAY-PAQUET, ALEXANDER ALEXANDROV, GUILLAUME CHAPUY and BERTRAND EYNARD.

TAMAS HAUSEL (IST VIENNA)

*Refined geometric invariants and representation theory.*

I will discuss two situations where refined geometric invariants of Higgs moduli spaces indicate the possibility of a refinement of the representation theory of finite groups and algebras of Lie type and affine Kač-Moody algebras respectively.

FELIX JANDA (MICHIGAN UNIVERSITY)

*Faber's tautological relations.*

More than 20 years ago, Carel Faber proposed a series of remarkable conjectures on the structure of the tautological ring of the moduli space of smooth curves. Most of his conjectures have been proven since then, and they have played a central role in the study of the tautological ring.

Faber found his conjectures by constructing a large set of relations between tautological classes of smooth curves. Later, Aaron Pixton proposed a description of the set of all relations between tautological classes of stable curves, but it is still very mysterious how these two sets of relations are connected.

In my talk, I want to discuss a conjectural extension of Faber's relations to the moduli space of stable curves.

EMMANUEL LETELLIER (INSTITUT MATHÉMATIQUE DE JUSSIEU)

*Counting geometrically indecomposable parabolic bundles on  $\mathbb{P}^1$ .*

I will discuss a formula for the counting of geometrically indecomposable parabolic bundles over the projective line over finite fields and explain the relationship with the cohomology of the moduli space of representations of the fundamental group of the punctured Riemann sphere with local monodromy in prescribed semi-simple conjugacy classes.

DINO LORENZINI (UNIVERSITY OF GEORGIA)

*Intersection matrices and Néron models.*

An intersection matrix is an integer matrix consisting in the adjacency matrix of a connected graph plus a strictly negative diagonal. Such matrices arise in algebraic geometry, for instance when studying the degenerations of curves. We will discuss how a purely combinatorial study of these matrices informs our understanding of the degeneration of the Jacobian of the curve (more precisely, of its Néron model), and in turn, how geometric ideas in the theory of curves motivate new results of combinatorial nature about these matrices.

HANNAH MARKWIG (TÜBINGEN UNIVERSITÄT)

*Counting curls and hunting monomials: Tropical mirror symmetry of elliptic curves.*

Mirror symmetry relates Gromov-Witten invariants of an elliptic curve with certain integrals over Feynman graphs. We prove a tropical generalization of mirror symmetry for elliptic curves, i.e., a statement relating certain labeled Gromov-Witten invariants of a tropical elliptic curve to more refined Feynman integrals. Joint work with JANKO BÖHM, ARNE BUCHHOLZ, KATHRIN BRINGMANN.

MARTIN MÖLLER (FRANKFURT UNIVERSITÄT)

*Volumes and Siegel-Veech constants for moduli space of flat surfaces I.*

The moduli space of abelian differentials carries a natural volume form due to Masur and Veech. Interest in these volumes and in Siegel-Veech constants stems from the dynamics on

billiard tables.

We provide an introduction to volumes and Siegel–Veech constants, and transform the problem into combinatorics and properties of quasimodular forms. This enables efficient computation and a proof of the large genus asymptotics conjecture for volumes and Siegel–Veech constants.

MOTOHICO MULASE (UC DAVIS)

*From Cayley to Hurwitz to Hitchin.*

B-model geometry captures quantum cohomology of A-model geometry. Since quantum cohomologies are further quantized by higher-genus Gromov-Witten theory, it is expected that B-model geometry should also have a unique quantization. One can ask, what does it mean to quantize a B-model geometry? In this talk, we start with a motivational example of classical tree counting problem. We then “quantize” tree counting, and observe that it becomes Hurwitz numbers. Via the Laplace transform, we can identify this quantization as construction of D-modules. Toward the end, we present our recent result of quantization in this context as a passage from families of Hitchin spectral curves to families of Rees D-modules. The talk is based on joint works with OLIVIA DUMITRESCU and PIOTR SUŁKOWSKI.

RAHUL PANDHARIPANDE (ETH ZÜRICH)

*Descendent invariants in the theory of stable pairs.*

I will discuss the construction and basic properties of descendent invariants in the theory of stable pairs on 3-folds. Descendents concern the Chern characters of the tautological sheaves and are parallel to descendents in the Gromov-Witten theory of 3-folds. I will discuss the GW/Pairs correspondence (joint work with AARON PIXTON), the analogs of the Virasoro constraints (joint work with ALEXEI OBLOMKOV and ANDREI OKOUNKOV), and the connection to the work of J. Shen on the virtual class in algebraic cobordism.

HUGO PARLIER (UNIVERSITÉ DU LUXEMBOURG)

*Interrogating length spectra and quantifying isospectral finiteness.*

Associated to a closed hyperbolic surface is its length spectrum, the set of the lengths of all of its closed geodesics. Two surfaces are said to be isospectral if they share the same length spectrum. There are different methods to produce surfaces that are isospectral but not isometric, the most successful one based on a technique introduced by Sunada. The talk will be about the following questions and how they relate:

- ★ How many questions do you need to ask a length spectrum to determine it?
- ★ Among all surfaces of given genus, how many can be isospectral but not isometric?

The approach to these questions will include finding adapted coordinate sets for moduli spaces and exploring McShane type identities.

BRAM PETRI (MPIM BONN)

*Random low-dimensional manifolds.*

The main question in this talk will be: what does a typical manifold look like? In order to study typical manifolds, we will consider random manifolds and ask what an average one looks like. I will discuss various models for random two- and three-dimensional manifolds and the geometry and topology of the manifolds they produce.

AARON PIXTON (MIT)

*Cycle-quasimodularity of elliptic curve invariants.*

Okounkov and Pandharipande proved that Gromov-Witten invariants of an elliptic curve are coefficients of quasimodular forms. I will explain how to lift this quasimodularity to the level of cycles (on the moduli space of stable curves). This is joint work with GEORG OBERDIECK.

KONSTANZE RIETSCH (KING'S COLLEGE)

*Grassmannians and polytopes.*

I will report on joint work with LAUREN WILLIAMS in which we use the mirror superpotential of a Grassmannian  $X$  (from a joint work with ROBERT MARSH) and the cluster tori in the mirror Grassmannian (for the cluster structure of Postnikov and Scott) to understand certain Newton-Okounkov convex bodies constructed using the network charts (of Postnikov and Talaska) on  $X$ .

DON ZAGIER (MPIM BONN)

*Volumes and Siegel-Veech constants for moduli space of flat surfaces II.*

In this talk, which is a companion to that of MARTIN MÖLLER, we will discuss the combinatorial and modular-forms aspects of the computations of volumes and Siegel-Veech constants mentioned in his abstract. A central role will be played by the Bloch-Okounkov theorem relating counting functions of ramified coverings of the torus to quasimodular forms, whose statement and proof will be sketched. This talk forms part of the master course I am currently giving at the IHP, but will be presented in an independent way.

ANTON ZORICH (INSTITUT MATHÉMATIQUE DE JUSSIEU)

*Random square-tiled surfaces, meanders, and Masur-Veech volumes.*

We show how recent equidistribution results allow to compute approximate values of Masur-Veech volumes of the strata in the moduli spaces of Abelian and quadratic differentials by Monte Carlo method. We also show how similar approach allows to count asymptotical number of meanders of fixed combinatorial type in various settings in all genera. Our formulae are particularly efficient for classical meanders in genus zero. Joint work with VINCENT DELECROIX, ELISE GOUJARD and PETER ZOGRAF.

## Poster session: Tuesday 14th, 6pm (IHP ground floor)

MARVIN ANAS HAHN (TÜBINGEN UNIVERSITÄT)

*Chamber behavior of mixed Hurwitz numbers*

Hurwitz numbers are enumerative objects counting functions between Riemann surfaces with fixed ramification data. Classical and monotone double Hurwitz numbers are two classes of Hurwitz-type counts of specific interest. Mixed double Hurwitz numbers are a combinatorial interpolation between those invariants. They are known to be piecewise polynomial. We give an algorithm involving tropical covers computing these polynomials in each chamber. Moreover, we use this approach to give wall-crossing formulas in genus zero.

LINXIAO CHEN (PARIS-SUD) & JOONAS TURUNEN (UNIVERSITY OF HELSINKI)

*Critical Boltzmann Ising triangulations of the half-plane*

We consider a random triangulation of the  $(p + q)$ -gon coupled to an Ising model on its faces, with a Dobrushin boundary condition  $+^p -^q$ . A Boltzmann weight is assigned to every face of the triangulation. This type of boundary condition is preserved by the simple peeling process that explores the Ising interface imposed by the boundary condition. We exploit this fact to compute explicitly the partition function of these random triangulations by solving the associated Tutte's equation. This partition function gives rise to a perimeter exponent different from that of a uniform triangulation for one unique value of the Ising coupling constant. We concentrate on this critical phase. Using exact asymptotics of the partition function, we show that in when  $p$  and  $q$  tend to infinity one after the other, the above random triangulation converges locally in distribution to an infinite triangulation of the half plane. Moreover, on this infinite triangulation there is essentially one unique infinite Ising interface, which touches the boundary only finitely often. A scaling limit result of cluster perimeter is also obtained.

ALESSANDRO MALUSÁ (QGM AARHUS)

*Quantisation in Chern-Simons theory via geometric constructions*

One approach to quantum Chern-Simons theory uses geometric quantisation applied to the moduli space of flat connections. For the theory with gauge group  $SL(n, \mathbb{C})$  the relevant Hilbert space is that of smooth sections of a suitable Hermitian line bundle over the moduli space of flat  $SU(n)$ -connections. By using the geometric properties of these space and the correspondences with other moduli spaces, one can associate to a wide family of classical observables tensor fields on the  $SU(n)$  moduli space. To these one can further associate differential operators on the Hilbert space, thus giving candidates for a quantisation. This poster presents the general ideas of these constructions and the ongoing investigation, specialised for now to the specific case of genus 1.

TIM REYNHOUT (CENTRAL MICHIGAN UNIVERSITY)

*Topological recursion for symplectic volumes of moduli of open Riemann surfaces.*

A brief background is provided including the definitions of spectral curve, ribbon graph and correlation functions as well as the relationship between the ribbon graph complex  $RG(g, n)$  and the Airy curve  $x = y^2$ . We show how to generalize these results to ribbon graphs with boundary.

JIFENG SHEN (YALE UNIVERSITY)  
*Lefschetz for non-Archimedean Jacobians.*

We establish a Lefschetz hyperplane theorem for the Berkovich analytifications of Jacobians of curves over an algebraically closed non-Archimedean field. Let  $J$  be the Jacobian of a curve  $X$ , and let  $W_d$  be the locus of effective divisor classes of degree  $d$ . We show that the pair  $(J^{\text{an}}, W_d^{\text{an}})$  is  $d$ -connected, and thus in particular the inclusion of the analytification of the theta divisor  $\Theta^{\text{an}}$  into  $J^{\text{an}}$  satisfies a Lefschetz hyperplane theorem for  $\mathbb{Z}$ -cohomology groups and homotopy group.