Workshop on Large Random Structures in Two Dimensions

Organizers: Marie Albenque and Cédric Boutillier

January 16-20 2017

- **Louigi Addario-Berry**(McGill University), *Random maps and random trees: limits, symmetries, and embeddings* .

  I will present and explain some recent results on random maps. The topics covered will include (a subset of): metric-space scaling limits; models of random growth; canonical embeddings and their limits; decompositions and connectivity; and symmetrizations of snakes.

  Parts of this are joint work with Marie Albenque, Nicholas Leavitt, and Yuting Wen.

- **Omer Angel** (University of British Columbia), *TBA.*

- **Jérémie Bouttier** (CEA, ENS Lyon), *The free boundary Schur process and applications.*

  The Schur process, originally introduced by Okounkov and Reshetikhin, is a random sequence of integer partitions whose transition rates are given by (skew) Schur functions. Several combinatorial models such as plane partitions, domino tilings of the Aztec diamond and last-passage percolation (LPP) can be recast as instances of Schur processes. Here we consider a variant of the Schur process involving free boundaries (in the original process the initial and final partitions are constrained to be empty). In the aforementioned applications, this corresponds to considering plane partitions, tilings or LPP with symmetries. The case of one free boundary (say, the initial one) was previously considered by Borodin and Rains.

  Our main result is a general expression for the correlation functions of the process, involving contour integrals suitable for asymptotics. Unlike the previously studied cases, the free boundary Schur process is neither determinantal nor pfaffian in general, but becomes pfaffian after "convoluting" it with an independent integer-valued random variable (this situation is reminiscent of the periodic Schur process introduced by Borodin). Our proof is based on the fermionic Fock space (aka semi-infinite wedge space) formalism.

  Work in progress with Dan Betea, Peter Nejjar and Mirjana Vuletić.

- **Timothy Budd** (Niels Bohr Institute), *On a connection between planar map combinatorics and lattice walks.*

  As a starting point I will present a combinatorial connection between walks on the upper half integer lattice and planar maps with controlled face degrees. The relation is shown to extend naturally to one between walks on the punctured plane and planar maps decorated with nested loops. The combinatorics of the latter has been studied in recent works by Borot, Bouttier, Duplantier, and Guitter in the setting of the (rigid) $O(n)$ loop model. I'll show how to use their generating functions to count walks on the (punctured) square lattice with control on the winding number around the origin. As an application one may obtain generating functions for walks on cones of arbitrary angles (integer multiples of $\pi/4$), which generalizes several known formulas in the combinatorics literature.

- **Alessandra Caraceni** (University of Bath), *Self-Avoiding Walks on Random Quadrangulations.*

  The local limit of random quadrangulations (UIPQ) and the local limit of quadrangulations with a simple boundary (the simple boundary UIHPQ) are two very well studied objects. We shall see how the simple boundary UIHPQ relates to an annealed model of self-avoiding walk on random quadrangulations, and how metric information obtained for the UIHPQ can be used to study quantities such as the displacement of the self-avoiding walk from the origin, as well as to ultimately investigate how the biasing of random quadrangulations by the number of their self-avoiding walks affects their local limit.

- **Guillaume Chapuy** (IRIF Paris), *Voronoï tessellations of Brownian maps, Painlevé-I recurrence, and bijections: an observation and a conjecture .*

  Take a Brownian map of genus g, pick 2 points "uniformly" at random, and let X, $1-X$ be the masses of the two Voronoi cells centered in those points. I will show that $E[X^2] = 1/3$, and this is true for ANY genus $g \geq 0$. Much more, I conjecture that the r.v. X is genus independent and is in fact... a uniform on $[0, 1]$. The same seems to be true for tessellations with $k \geq 2$ points. This conjecture is open even for genus zero!

  This problem arises when one tries to enumerate maps of genus g via the state-of-the-art bijections (Marcus-Schaeffer/Miermont). One encounters a difficult subcounting problem that can be interpreted in terms of Voronoi tessellations and the quantity $E[X^2]$ naturally shows up in the calculations. By comparing the output with the known "tg-recurrence" (a Painleve-I recurrence which governs the asymptotic number of maps of genus g and large size, classically obtained by the double-scaling limit of the 1-matrix model and strongly linked to the integrable structure of the problem) we find, with surprise and frustration, that $E[X^2]$ is $1/3$ and in particular is genus independent. Something so strong comes either from something deep or from a simple trick, and in both cases I would be happy to know!

- **Mihai Ciucu** (Indiana University), *Lozenge tilings with gaps in a 90 degree wedge domain with mixed boundary conditions.*

  We consider a triangular gap of side two in a 90 degree angle on the triangular lattice with mixed boundary conditions: a constrained, zig-zag boundary along one side, and a free lattice line boundary along the other. We study the interaction of the gap with the corner as the rest of the angle is completely filled with lozenges. We show that the resulting correlation is governed by the product of the distances between the gap and its three images in the sides of the angle. This provides evidence for a unified way of understanding the interaction of gaps with the boundary under mixed boundary conditions, which we present as a conjecture. Our conjecture is phrased in terms of the steady state heat flow problem in a uniform block of material in which there are a finite number of heat sources and sinks. This new physical analogy is equivalent in the bulk to the electrostatic analogy we developed in previous work, but arises as the correct one for the correlation with the boundary.

  The starting point for our analysis is an exact formula we prove for the number of lozenge tilings of certain trapezoidal regions with mixed boundary conditions, which is equivalent to a new, multi-parameter generalization of a classical plane partition enumeration problem (that of enumerating symmetric, self-complementary plane partitions).

- **Hugo Duminil-Copin** (Geneva University), *Order-disorder operators in the critical Ising model on planar and almost planar graphs.*

  In this talk, we discuss boundary spin correlations of almost planar graphs satisfy Pfaffian formulas in the scaling limit. The proof is based on relations between different graphical representations of the Ising model, including the random-current and random-cluster ones.

- **Erik Duse** (University of Helsinki), *The Cusp-Airy Process.*

  In this talk we will discuss some aspects of the derivation of the Cusp-Airy process for uniformly random discrete interlacing models. We will also compare this with the derivation of the extended Airy process for the same random model.

- **Patrick Ferrari** (Bonn University), *Anisotropic (2+1)-d growth and Gaussian limits of q-Whittaker processes.*

  We consider a discrete model for anisotropic (2+1)-dimensional growth of an interface height function. Owing to a connection with q-Whittaker functions, this system enjoys many explicit integral formulas. By considering a certain limit, we obtain a Gaussian process and determine its space-time covariance. We see the phenomenon of slow-decorrelation along characteristic directions. and if we look on a smaller spatial scale we obtain a space-time limit to the (2+1)-dimensional additive stochastic heat equation (or Edwards Wilkinson equation) along characteristic directions. In particular, the bulk height function converges to a Gaussian free field which evolves according to this stochastic PDE.

- **Adrien Kassel** (ENS Lyon), *Active spanning trees and Schramm–Loewner evolution.*

  I will talk about a family of random space-filling curves on planar maps which is believed to converge to the Schramm–Loewner evolution (SLE_κ) in the scaling limit for all values of the diffusivity parameter $\kappa \geq 0$. A subfamily consists of the exploration curves for the Fortuin–Kasteleyn model which have been proven to converge to SLE in two integrable cases ($\kappa = 8$ and $\kappa = 16/3$). When the map is itself random, the model becomes integrable for all values of κ. This allows to show a scaling limit result, which based on recent breakthroughs in the understanding of SLE-decorated random surfaces, can be interpreted as the convergence of these discrete curves to SLE for all values of κ in the so-called peanosphere topology.

  Joint work with David B. Wilson (Microsoft), and with Ewain Gwynne (MIT), Jason P. Miller (Cambridge), and David B. Wilson.

- **Richard Kenyon** (Brown University), *Linear spaces of tilings.*

  We discuss spaces of tilings of planar regions with convex polygons; we give parameterizations of families of such tilings in terms of harmonic functions on planar networks; these parameterizations allow us to understand the topology of these spaces, and give natural probability measures on them.

- **Benoît Laslier** (Cambridge University), *Universality of height fluctuations in the dimer model.*

  I will present a new approach to the study of height fluctuations in the dimer model, not writing any determinantal formula. The key idea will be to make precise the fact that the imaginary geometry coupling between SLE and GFF is the continuous limit of the bijections between dimers and spanning, and to use the convergence of loop-erased random walk to SLE. We will obtain convergence of the height in many settings, including lozenge tilings of "Temperleyan like" planar domains of arbitrary slope and global shape.

- **Zhongyang Li** (University of Connecticut), *Phase transitions in the 1-2 model.*

  A configuration in the 1-2 model is a subgraph of the hexagonal lattice, in which each vertex is incident to 1 or 2 edges. By assigning weights to configurations at each vertex, we can define a family of probability measures on the space of these configurations, such that the probability of a configuration is proportional to the product of weights of configurations at vertices.

  We study the phase transition of the model by investigating the probability measures with varying weights. We explicitly identify the critical weights, in the sense that the edge-edge correlation decays to 0 exponentially in the subcritical case, and converges to a non-zero constant in the supercritical case, under the limit measure obtained from torus approximation. These results are obtained by a novel measure-preserving correspondence between configurations in the 1-2 model and perfect matchings on a decorated graph, which appears to be a more efficient way to solve the model, compared to the holographic algorithm used by computer scientists to study the model.

  When the weights are uniform, we prove a weak mixing property for the finite-volume measures - this implies the uniqueness of the infinite-volume measure and the fast mixing of a Markov chain Monte Carlo sampling. The major difficulty here is the absence of stochastic monotonicity.

- **Svante Linusson** (KTH Stockholm), *Reverse juggling .*

  I will describe a Markov chain model where a state is a matrix over a finite field. In each transition a new column is added to the left. This model has natural connections to what we call reverse juggling and which was recently studied by Knutsen. In joint work with Arvind Ayyer we generalise this in several ways.

- **Irène Marcovici** (IECL Nancy), *Does Eulerian percolation on the square lattice percolate?*

  Eulerian percolation on the square lattice with parameter p is the classical Bernoulli bond percolation with parameter p, conditioned on the fact that every site has an even degree. We first explain why Eulerian percolation with parameter p coincides with the contours of the Ising model for a well-chosen parameter $\beta(p)$. Then, we study the percolation properties of Eulerian percolation. Some key ingredients of the proofs are couplings between Eulerian percolation, the Ising model, and FK-percolation.

  This is a joint work with Régine Marchand and Olivier Garet.

- **Grégory Miermont** (ENS Lyon), *On the geometry of random surfaces with a boundary.*

  We classify the possible local and scaling limits of random maps with a boundary. This results in an interesting "phase diagram" parametrized by the ways in which the volume and perimeter of the maps are allowed to go to infinity. In passing, we find new families of Brownian surfaces, including an "infinite volume Brownian disk", and more importantly, a one-parameter family of half-planar random spaces that interpolate between the so-called "Brownian half plane" and Aldous' self-similar CRT. We will also compare and contrast the properties of geodesic rays in the half-planar local limit (where we show that geodesic rays intersect the boundary infinitely many often) and scaling limits (where geodesic rays avoid the boundary).

  Based on joint work with Erich Baur, Jérémie Bettinelli and Loïc Richier

- **Sevak Mkrtchyan** (Rochester University), *The GUE corners process for volume measures on lozenge tilings.*

  In the thermodynamic limit of the lozenge tiling model the frozen boundary develops special points where the liquid region meets with two different frozen regions. These are called turning points. It was conjectured by Okounkov and Reshetikhin that in the scaling limit of the model the local point process near turning points should converge to the GUE corners process. We will discuss a joint result with L.Petrov establishing the GUE corners process when the underlying measure is the "homogeneous q to the volume" measure. We'll also see how this process is modified when weights are not homogeneous anymore.

- **Peter Nejjar** (IST Austria), *Fluctuations of the Competition Interface in Presence of Shocks.*

  We consider last passage percolation (LPP) models with exponentially distributed random variables, which are linked to the totally asymmetric simple exclusion process (TASEP). The competition interface for LPP was introduced by P. Ferrari and Pimentel for cases where the corresponding TASEP had a rarefaction fan. Here we consider situations with a shock and determine the law of the fluctuations of the competition interface around its deterministic law of large number position. We also study the multipoint distribution of the LPP around the shock, extending our previous one-point results.

  Joint work with P. L. Ferrari

- **Jonathan Novak** (University of California), *Lozenge tilings, HCIZ integrals, and Hurwitz numbers.*

  I will present a method for obtaining limit laws for uniformly random lozenge tilings of certain planar domains which is based on the asymptotic analysis of Harish-Chandra/Itzykson+Zuber integrals. The HCIZ integral admits an asymptotic expansion in which certain generalized Hurwitz numbers play the role of Feynman diagrams, and one sees in this way new connections between dimers and Hurwitz theory. As an application, I will use this method to explain why the joint distribution of tiles of one type along a fixed height cross-section through a random tiling becomes, after rescaling, the spectrum of a fixed size GUE random matrix.

- **Greta Panova** (University of Pennsylvania), *Lozenge tilings: from combinatorics, through algebra, to probability.*

  Lozenge tilings arise in statistical mechanics/probability as dimer models on the hexagonal grid, but they are also plane partitions and Gelfand-Tsetlin patterns in algebra/combinatorics. We will discuss how they can be studied with the help of Schur generating functions, and when the formulas are "nice" can be studied asymptotically to derive the behavior of these tilings when the mess size goes to 0. In particular, we will show how to obtain GUE-corners process and limit surface (LLN) in tilings with global symmetries. We will also discuss tilings with multivariate weights arising from Schubert calculus.

  The talk will include results based on joint works with V.Gorin, and A.Morales, I.Pak.

IT+01	lun. 16/1	mar. 17/1	mer. 18/1	jeu. 19/1	ven. 20/1
09:00	**09:30** - Coffee & Registration	**09:30** - Coffee & Pastries	**09:30 – 10:20** Richard Kenyon	**09:30** - Coffee & Pastries	**09:30 – 10:20** Adrien Kassel
10:00	**10:00 – 10:50** Grégory Miermont	**10:00 – 10:50** Omer Angel	**10:20** - Coffee & Pastries **10:50 – 11:40** Mihai Ciucu	**10:00 – 10:50** Hugo Duminil-Copin	**10:20** - Coffee & Pastries **10:50 – 11:40** Peter Nejjar
11:00	**11:00 – 11:50** Alessandra Caraceni	**11:00 – 11:50** Guillaume Chapuy	**11:40 – 12:30** Jonathan Novak	**11:00 – 11:50** Timothy Budd	**11:40 – 12:30** Louigi Addario-Berry
12:00					
13:00					
14:00	**14:00 – 14:50** Irène Marcovici **14:50 – 15:40** Benoît Laslier	**14:00 – 14:50** Sevak Mkrtchyan **14:50 – 15:40** Greta Panova		**14:00 – 14:50** Patrick Ferrari **14:50 – 15:40** Erik Duse	
15:00	**15:40** - Coffee & break	**15:40** - Coffee break		**15:40** - Coffee break	
16:00	**16:10 – 17:00** Svante Linusson	**16:10 – 17:00** Jérémie Bouttier		**16:10 – 17:00** Zhongyang Li	
17:00					
18:00		**18:00 – 19:30** Cocktail at IHP			
19:00					